

# Secret Student Seminar

Experimental Algebra & Geometry Lab

## The Banach-Tarski Paradox and Paradoxical Decompositions in $\mathbb{R}^2$ and $\mathbb{R}^3$

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### Abstract

The Banach-Tarski Paradox allows one to take a ball in space, partition it into a finite number of pieces and rearrange them via Euclidean isometries yield a pair of balls each the same size as the original. In this talk, we will prove this famous theorem by using the Axiom of Choice and constructing a free subgroup in  $SL_3(\mathbb{R})$  of rank 2. Important concepts such as paradoxical sets and congruence with respect to Euclidean motions will be introduced as we go along. After the proof, we will examine the conditions under which paradoxical decompositions with respect to linear transformations exist in the plane, concluding with a sketch of von Neumann's paradox.

Date: Friday, November 15, 2013

Time: 2:00pm–3:00pm

Place: MAGC 1.302

**Pizza and soda will be served at the presentation.**